1) a) \( r = 0.05 \, m \)  
\( T = 2.5 \, s \)  
\( \frac{T}{2 \pi} = \frac{2.5}{2 \pi} \)  
\( w = \frac{1}{T} \)  
\( V_{\text{max}} = r \omega \)  
\( = (0.05 \times \pi) \)  
\( = 0.157 \, m/s \)  

b) \( a = -\omega^2 x = -\pi^2 (0.025) = -0.2 \times 7 \, m/s^2 \)

2) period = 3.5  
\( f = \frac{1}{3.5} \)  
\( w = \frac{2\pi}{3} \, s^{-1} \)  
\( a = 15 \, mm \)  
\( \frac{\pi}{2} \)  
\( \sin (\omega t + \phi) = 1 \)  
\( \sin \phi = 1 \)  
\( \Rightarrow \phi = \frac{\pi}{2} \)

3) Amplitude \( A = \frac{30-10}{2} = 10 \) division  
\( T = 1.5 \, s \)  
\( \frac{2\pi}{T} = \frac{2\pi}{1.5} \)  
\( x = a \sin (\omega t - \frac{\pi}{2}) \)  
18 division \( \Rightarrow x = -2 = 10 \sin \left(\frac{2\pi}{1.5} t - \frac{\pi}{2}\right) \)  
25 division \( \Rightarrow x = 5 = 10 \sin \left(\frac{2\pi}{1.5} t - \frac{\pi}{2}\right) \)

From (1):  
\( \frac{\pi}{2} t_1 - \frac{\pi}{2} = -0.06 \, \pi \)  
\( \frac{\pi}{3} t_1 = 0.5 - 0.06 \)  
\( = 0.44 \, s \)  
\( t_1 = 0.33 \, s \)

From (2):  
\( \frac{\pi}{2} t_2 = 0.5 - 0.25 \)  
\( \frac{\pi}{6} t_2 - \frac{\pi}{2} = 0.25 \)  
\( t_2 - t_1 = 0.175 \)
At Class Ex6

\[ \frac{1}{2} R (2.0)^2 - \frac{1}{2} R (4.0)^2 = \frac{4 - 16}{16} = -\frac{12}{16} = -\frac{3}{4}. \]

5. Ke. when passing 2nd pt.: Ke. = \( \frac{1}{2} R A^2 \)

\[ = \frac{1}{2} m w^2 A^2 = \frac{1}{2} (0.002) (\frac{2\pi}{2})^2 (0.01^2) = 9.87 \times 10^{-5} \text{ J}. \]

Ke. when passing 3rd pt.: Ke. = \( \frac{1}{2} R (A^2 - X^2) \)

\[ = \frac{1}{2} (0.002) (\frac{2\pi}{2})^2 (0.01^2 - 0.01^2) = \frac{1}{2} (0.002) \pi^2 (0.0075) = 7.40 \times 10^{-5} \text{ J}. \]

6. (a) Extension \( x = 0.35 \text{ m} \)

\[ F = kx \]

\[ k = \frac{1.5}{0.055} = 27.3 \text{ N m}^{-1} \]

(b) \[ T = 2\pi \sqrt{\frac{\mu}{27.3}} \]

\[ = 2\pi \sqrt{\frac{0.15}{27.3}} = 0.4665 \text{ (r = amplitude)} \]

(c) \[ V_{\text{max}} = R W = (0.03) \left( \frac{2\pi}{0.466} \right) = 0.404 \text{ m s}^{-1} \]

\[ \text{Ke.} = \frac{1}{2} m v^2 = \frac{1}{2} (0.15)(0.4)^2 = 0.125 \]

(d) Max tension at lowest pt.

Restoring force \( F = T - mg = ma \).

\[ a = \frac{V_{\text{max}}^2}{R} = \frac{0.404^2}{0.03} = 53.6 \text{ m s}^{-2} \]

\[ T = mg + \mu \]

\[ = 2.3 \text{ N} \]

Min tension at highest pt.

Restoring force \( F = mg - T = ma \).

\[ T = mg - R \]

\[ = 0.62 N \]
At Class 7C-6

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(a) At \( x = 0.2 \text{ m} \), the potential energy maximum and is equal to 2 J.

By \( E_p = \frac{1}{2}kx^2 \), the force constant is

\[
k = \frac{2E_p}{x^2} = \frac{2 \times 2}{0.2^2} = 100 \text{ N m}^{-1}
\]

(b) At \( x = 0.1 \text{ m} \), the elastic p.e. of the spring

\[
E_p = \frac{1}{2}kx^2 = \frac{1}{2} \times 100 \times 0.1^2 = 0.5 \text{ J}
\]

Since the total energy is 2 J, at \( x = 0.1 \text{ m} \), the kinetic energy is

\[
E_k = U_0 - E_p = 2 - 0.5 = 1.5 \text{ J}
\]

The speed is given by

\[
E_k = \frac{1}{2}m v^2 \Rightarrow v = \sqrt{\frac{2 \times 1.5}{0.25}} = 3.46 \text{ m s}^{-1}
\]

Alternatively, let’s find the angular frequency

\[
\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{100}{0.25}} = 20 \text{ rad s}^{-1}
\]

By \( v = \omega \sqrt{A^2 - x^2} \), the speed is

\[
v = 20 \times \sqrt{0.2^2 - 0.1^2} = 3.46 \text{ m s}^{-1}
\]

(c) Since \( \omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{100}{0.25}} = 20 \text{ rad s}^{-1} \),

maximum acceleration is

\[
a_0 = A \omega^2 = 0.2 \times 20^2 = 80 \text{ m s}^{-2}
\]

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(a) \( F = kx \)

\[
k = \frac{2 \times 5}{0.05} = 50 \text{ N m}^{-1}
\]

(b) \( T = \frac{2\pi}{\frac{\sqrt{m}}{k}} \)

\[
= 2\pi \sqrt{\frac{0.05 \times 0.84}{50}} = 0.845
\]

Time required \( = \frac{T}{2} = 0.425 \text{ s} \).

Speed passes equal position

\[
v = r \omega \]

\[
= (0.05) \times \left( \frac{2\pi}{0.84} \right)
\]

\[
= 0.373 \text{ m s}^{-1}
\]

(c) Period \( T \) unchanged.

Time required = 0.425 s

Speed = \( r \omega \)

\[
= (0.05) \times \left( \frac{2\pi}{0.84} \right)
\]

\[
= 0.166 \text{ m s}^{-1}
\]

(d) \( T' = 0.84 \times 2 = 1.68 \text{ s} \)

\[
1.68 = 2\pi \sqrt{\frac{m'}{50}}
\]

\[
m' = 3.57 \text{ kg}
\]

Additional mass = 3.57 - 0.9

\[
= 2.67 \text{ kg}
\]
(a) (i) Time for 1 oscillation = 1.25
\[ f = \frac{1}{T} = 0.8 \text{ Hz} \]

(ii) They have same period
\[ T \propto \sqrt{\frac{1}{k}} \]
equivalents \( m \) equals

(iii) show that the variation is a sine curve.

(c) P.E. = \( \frac{1}{2} kx^2 \)
\[ \frac{1}{2} k(0.2)^2 = 0.16 \text{ J} \]
\[ k = 8 \text{ Nm}^{-1} \]

(b) By \( \Gamma = c \theta \), the restoring couple is
\[ \Gamma = 2 \times 10^{-3} \times 0.1 = 2 \times 10^{-4} \text{ Nm} \]

(b) The moment of inertia of the sphere about the axis is
\[ I = \frac{2}{3} MR^2 = \frac{2}{3} \times 0.2 \times 0.05^2 = 2 \times 10^{-4} \text{ kg m}^2 \]

By \( T = 2\pi \sqrt{\frac{I}{c}} \), the period of oscillation is
\[ T = 2\pi \sqrt{\frac{2 \times 10^{-4}}{2 \times 10^{-3}}} = 1.98 \text{ s} \]

(a) The moment of inertia of the rod about the pivoted end is
\[ I = \frac{1}{3} ML^2 = \frac{1}{3} \times 0.27 \times 1^2 = 0.09 \text{ kg m}^2 \]

(b) The weight is at the mid-point of the rod.

\[ \sin \theta = \frac{2.7 \times 1}{2} \sin \theta = 1.35 \sin \theta \]

(c) By \( \Gamma_{net} = I \alpha = 0.09 \theta \), the angular acceleration is
\[ \theta = \frac{1.35}{0.09} \sin \theta = -15 \sin \theta \]

If the amplitude of swing is small, we have \( \sin \theta = \theta \). Thus,
\[ \theta = -15 \theta \]

This represents SHM with angular frequency \( \omega = \sqrt{15} = 3.87 \text{ rad s}^{-1} \) and its period is
\[ T = \frac{2\pi}{\omega} = \frac{2\pi}{3.87} = 1.62 \text{ s} \]